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THE OBLIQUITY OF THE ECLIPTIC TWO MILLENIA AGO

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SUMMARY

Hellenistic measurements of star declinations allow us to infer the obliquity of the ecliptic at the mean epoch −114. The result is $23°\,43'\,06'' \pm 3'\,19''$. The value calculated from Newcomb's theory at the same epoch is $23°\,42'\,35''$.

1. INTRODUCTION

Newcomb's expression for the mean obliquity ϵ of the ecliptic is

$$\epsilon = 23°{\cdot}452\,294 - 0°{\cdot}013\,012\,5T \\ - 0°{\cdot}000\,001\,64T^2 + 0°{\cdot}000\,000\,503T^3, \tag{1}$$

in which T is time in Julian centuries from the epoch 1900 January 0.5. In recent years, two questions have been raised about the accuracy of this expression. Duncombe & Clemence (1958), on the basis of measurements covering about the last century, suggest that an amount of about $-0''{\cdot}25T$ needs to be added to the right member of equation (1). Needham (1959) summarizes a number of ancient and medieval measurements of ϵ and finds that measurements before the year 0 tend to be larger than the value from equation (1) by 10′ or more. As a result, he suggests that equation (1) may need revision. The change that is implied by Needham's data is much larger in magnitude than the change suggested by Duncombe and Clemence, but it is in the same direction.

For example, Ptolemy (*c* 142, Chapter I.12) quotes a measurement made by Eratosthenes about −250, who found $\epsilon = 23°\,51'\,20''$. Equation (1) gives $23°\,43'\,34''$ for this epoch, so that the disagreement is about 8′, or about half the semi-diameter of the Sun. The gnomon, which was a popular instrument in Eratosthenes' time, casts an asymmetrical shadow containing both an umbra and a penumbra onto a horizontal plane. If Eratosthenes used a gnomon in his measurement, an error of half the semi-diameter of the Sun is not surprising.

In studying old measurements of the obliquity, we are hampered by large measurement errors and by the occurrence of two hoaxes, one ancient and one medieval. The Muslim algebraist and astronomer Abu Sahl al-Kuhi committed the medieval one. al-Biruni (1025) quotes him as claiming that he had measured the meridian altitude of the Sun at both solstices from Baghdad, and that he had found $23°\,51'\,20''$ for the obliquity and $33°\,41'\,20''$ for the latitude of Baghdad. The error in the latitude is about 21′. Further, as al-Biruni points out, the obliquity disagrees by about 16′ with all other measurements made at the same time, which was near 990. The errors in both values equal or exceed the semi-diameter of the Sun and are virtually impossible to make. However, the claimed obliquity agrees to the second of arc with the value found by Eratosthenes. There can be little doubt that al-Kuhi's claimed measurements were a hoax.

The ancient hoax was committed by Ptolemy himself (Newton 1973). In Chapter I.12 of his work, Ptolemy (*c*. 142) describes two instruments for measuring the altitude of the Sun, one that he did not use and one, a mural quadrant, that he says he did use several times for measuring the obliquity. Since the quadrant instrument makes a symmetrical shadow, there should have been no bias in Ptolemy's result. Further, since the obliquity decreases and Ptolemy's measurements were four centuries after Eratosthenes, Ptolemy should have found a much smaller value than Eratosthenes. In spite of these facts, Ptolemy agrees with Eratosthenes to the second of arc. The odds that Ptolemy's result is a hoax have been estimated (Newton 1973) at 10^{92} to 1.

Fortunately, we are not dependent upon ancient measurements of the obliquity in estimating what its value was in ancient times. Two ancient sources contain a large body of data from which we may make an independent estimate for an epoch near −100.

2. THE ANCIENT SOURCES

In his Chapter VII. 3, Ptolemy lists the declinations of 18 stars as measured by Timocharis or Aristyllus around the year −290, by Hipparchus about −128, and by himself about +137. Ptolemy makes no use of the declinations of 12 of the stars, but he uses the declinations of six stars in order to derive a rate of 36″ per year for the precession of the equinoxes. It has been shown (Newton 1974b) that the declinations which Ptolemy claims to have measured himself are valid data for the 12 stars that he did not use, but that they are fraudulent for the six that he did use. The odds in favour of this conclusion are greater than 3000:1. One way to reach the conclusion is to derive a precession rate from the 12 stars that he did not use. The result is $52''\cdot 8 \pm 2''\cdot 0$ per year, in good agreement with the value derived from modern data and in clear disagreement with the value 36″ per year. The odds are small that Ptolemy by accident happened to use those data and only those data that lead to 36″ per year.

There is no reason to question the authenticity of the values attributed to Timocharis, Aristyllus, and Hipparchus, and they have the appearance of genuine data.

Thus Ptolemy has preserved 54 values of the declinations of stars at epochs from −290 to +137. Forty-eight of the values represent valid observations while six are fraudulent.

Around −380, the astronomer Eudoxus of Cnidus, who had studied both in Athens and in Egypt, wrote a book about the appearance of the heavens. A century later, the poet Aratus of Soli wrote what is variously called a rendering or an imitation of Eudoxus's work in verse. Both of these works are lost except for a few fragments of Aratus's poem and for Latin translations of it. A century and a half after Aratus, Hipparchus (*c* −128) wrote a book of commentary on the works by Eudoxus and Aratus. This is the only work of Hipparchus that survives.

In this work, Hipparchus typically quotes a short passage from Eudoxus or Aratus or both. He then comments on the passage, usually to show that it is not correct. For example, in a discussion of the circle of perpetual occultation, Hipparchus (pp. 114–115) quotes Eudoxus as saying: 'Canopus, which is visible in Egypt, lies on the same'. Hipparchus comments that this is not correct. He says that the circle lies 36° from the south pole in Rhodes and that it lies 37°

from the pole in Athens, while Canopus is about $38\frac{1}{2}$ degrees from the south pole. Hence, says Hipparchus, Eudoxus is in error; Canopus is above the circle of perpetual occultation even in Greece.*

Altogether, I found 40 declincations of stars that Hipparchus uses in commenting upon Aratus and Eudoxus. Some of the values are stated directly as declinations (distances from the equator). More values are given in terms of the distance from one of the poles; I shall convert these to declinations without comment. Four values are given in terms of distance from the summer solstice. Since locating the solstice is the purpose of this study, there is no way to use those values here. This leaves 36 declinations.

Three stars in the list given by Ptolemy are also used by Hipparchus. These are α Boötis, α Geminorum, and β Geminorum. For α Boötis and β Geminorum, the values given by Hipparchus agree with those which Ptolemy says that Hipparchus measured. This confirms, if confirmation were needed, that the declinations given by Hipparchus were measured by himself. For α Geminorum, Hipparchus gives $33\frac{1}{2}$ degrees while Ptolemy says that he measured $33\frac{1}{6}$ degrees. With certain forms of Greek numerals, it is easy to confuse $\frac{1}{6}$ and $\frac{1}{2}$, and this is probably not a genuine difference. It does not matter much which value we use, and I shall use the value listed by Ptolemy.

I shall keep these three stars in the Ptolemy list and remove them from the Hipparchus list. This leaves 33 declinations to use from the writing of Hipparchus.

There are problems with some of Hipparchus's values. In most places, it seems to be his intention to give precise values, but in a few places, where he does not need precision, he seems to be approximate. He says that β and ι Ophiuchi are about $7°$ and $15°$, respectively, from the equator. He says that τ and g Piscium have the same declination, when in fact they differ by considerably more than $1°$. I shall assume that the errors in these statements are twice the standard error of measurement and hence I shall give these values relative weights of $\frac{1}{4}$.†

In several cases, Hipparchus says that a value is slightly greater than some number; for example, he says that the declination of η Ophiuchi is somewhat greater than $10°$ south. I add $0°.2$ to the values stated in these cases, which will be identified in the table to be given later. An exception concerns γ Pegasi. Hipparchus says that its declination is more than $3\frac{1}{2}$ degrees; I take this to be $3°.6$. A reasonable error in these values has no significant consequence.

With regard to σ Andromedae, the Greek text in Manitius's edition says that it is more than $30°$ north of the equator, while the translation says 'more than $25°$'. The correct declination is slightly more than $25°$, so that there is apparently a typographical error in the text. I shall assume that Hipparchus originally wrote $25°$.

* Here it seems that Eudoxus is more accurate than Hipparchus. For Athens, whose latitude is $38°.0$, the true polar distance of points on the apparent circle of perpetual occultation is about $37°.4$, while the polar distance of Canopus (α Carinae) was about $37°.3$ in Hipparchus's time. Thus Canopus was almost exactly on the circle in Athens. The place where Eudoxus wrote does not matter for the purposes of this work. It was clearly not Egypt, and Athens is the most likely alternative.

† Hipparchus does not use designations like β Ophiuchi, but there is no problem in identifying his stars, and I shall give only the modern designations. Manitius gives identifications in the cited translation of Hipparchus. I identified the stars independently and found no discrepancy.

Hipparchus says that μ Draconis is $34\frac{3}{5}$ degrees from the pole, but the correct distance is close to $31\frac{3}{5}$ degrees. It is easy to confuse the digits '1' and '4' in Greek numerals, and it is plausible either that Hipparchus mistook '1' for '4' in copying from his notes or that a later copyist made this mistake. For safety, I shall give this observation a weight of zero in the analysis, although I shall carry through all the needed calculations for it so that others may use it if they wish.

Hipparchus says that the declination of ϵ Lyrae is 41°, but it was actually close to 39° in his time. I have not thought of a simple explanation of this error, but it is extremely unlikely to be an error of observation. I shall perform the calculations for this observation while giving it a weight of 0.

Hipparchus says that the declination of the most westerly of the three stars in the tail of the northern fish (ψ^1 Piscium) is 12°, but the correct value is about 9°·7. However, the declination of the most easterly of the three (ψ^3 Piscium) is close to the stated value. It is easy to interchange 'easterly' and 'westerly' by accident in writing or speaking, and it is plausible that Hipparchus meant to identify ψ^3 Piscium. Again I shall perform the calculations using the stated value, to which I shall give a weight of zero.

3. THE METHOD OF ANALYSIS

Since the stars to be used lie both north and south of the observer, the effects of refraction tend to cancel. However, it is simple to apply a correction for refraction, and I shall do so by assuming that the declination of a star was measured when it was at culmination. There is no direct evidence that this was done, but it is simple and it is probably the most likely method for an ancient astronomer to use. Use of the assumption does decrease the residuals slightly without changing the value inferred for the obliquity by an appreciable amount.

The refraction correction is calculated as a function of the apparent zenith distance, using tables (Allen 1962) that apply at a temperature of 10 C and a pressure of 760 mm.

The relation to be used in inferring the obliquity is

$$\sin\delta = \cos\beta\sin\lambda\sin\epsilon+\sin\beta\cos\epsilon. \tag{2}$$

In this, δ is the observed declination (after correction for refraction), β is the latitude, and λ is the longitude. In order to use equation (2), we must have values of λ and β at the epoch of the measurement. We can then make an estimate of ϵ from each measurement by converting equation (2) into a quadratic in $\sin\epsilon$ and solving. This gives us about 80 individual estimates of ϵ.

In finding a mean value of ϵ from the individual estimates, we note that the weight given to an individual estimate should depend upon the location of the star. Let $\Delta\delta$ denote the error in a measurement of δ and let $\Delta\epsilon$ denote the error in the value of ϵ deduced from δ, assuming for the moment that β and λ are known exactly. The relative weight that an individual estimate should receive is then equal to $(\Delta\delta/\Delta\epsilon)^2$. From equation (2), we see that this is simply $\sin^2\alpha$, in which α is the right ascension.

I shall show in a moment that the values of β and λ to be used in equation (2) should be taken from modern data. Since the errors in these are negligible compared with those in the ancient measurements of δ, the preceding error analysis is valid.

There is a problem about the values of λ and β to use in solving equation (2). The natural thought is to use the anciently measured values of λ and β as well as of δ. However, the only ancient values of λ and β that we have are those found in Books VII and VIII of the *Almagest* (Ptolemy, *c* 142). Now we know that Ptolemy 'proved' that the precession of the equinoxes is 36″ per year by two methods. He used the fraudulent declinations of six stars that were mentioned in the preceding section (see also Newton 1974b). He also used a direct measurement (Chapter VII.2) of the longitude of Regulus (α Leonis). He entered the result of this measurement as the longitude of Regulus in his star table. He also compared it with the longitude measured by Hipparchus 265 years earlier and again 'proved' that the precession is 36″ per year. It has been shown clearly (Newton 1974a) that Ptolemy's observation of Regulus is fraudulent.

In 265 years, the total precession at 36″ per year is $2\frac{2}{3}$ degrees, which is too small by slightly more than 1°. We know that all the longitudes in Ptolemy's star table are too small by about 1°. This is just what we should expect if the longitudes had been fudged by adding $2\frac{2}{3}$ degrees to the longitudes in the now-lost star table of Hipparchus. We also know by direct evidence that the longitude of one star in Ptolemy's table was fudged, and the most likely method of fudging was to add $2\frac{2}{3}$ degrees to Hipparchus's value. Thus there is a strong presumption that we can restore Hipparchus's table by simply subtracting $2\frac{2}{3}$ degrees from the longitudes and leaving the latitudes alone.

This presumption is attractive. However, it seems to me that it is not as well established as is the existence of fudging on Ptolemy's part, and hence it is not safe to use it in the inference of the obliquity. If we do not use it, we have no alternative to using values of λ and β calculated from modern data. Calculation of λ and β for a star at an ancient epoch, including the proper motion, is straightforward and it does not seem necessary to give details.

Use of calculated values of β raises a question. Most of the linear term in equation (1) comes from the rotation of the ecliptic. Since the rotation of the ecliptic also enters into the calculation of ancient latitudes, we need to ask if we are reasoning in a circle if we use values of β calculated from modern data.

The answer to this can be found by looking upon equation (2), in combination with the transformation between modern ecliptic coordinates and ancient ones, as a complicated relation for the rate of rotation of the ecliptic. We can solve relations of this sort by successive approximations. We assume a rate from Newcomb's theory and use it in calculating the latitudes for insertion into equation (2). We then solve equation (2) for the obliquity and hence for the rate of rotation. If necessary, we use the new rate in a new calculation of the ancient latitudes, and so on until the process converges.

Some stars listed as single in ancient writing are actually close doublets. In these cases, I use the average position of the two stars in the doublet, unless one of the two is much fainter than the other. When this happens, I use only the bright component.

4. INSTRUMENTAL BIAS

Even after we correct the quoted values of declination for refraction, we still expect to find an instrumental bias in the declination because the observers could probably not find the local vertical with an accuracy better than a few minutes of arc. Since four observers are involved in the measurements of declination, and since it is unlikely that they had the same bias, the effects of bias tend to disappear from the inference of the obliquity.

Further, if the bias in a given instrument is independent of the right ascension of the star being observed (which means that the bias must be independent of the seasons), the bias does not give rise to an error in the inferred obliquity, provided that the stars used are suitably distributed in right ascension or longitude. In order to see this, consider a star with $\alpha = 90°$, with latitude β_1, and with declination δ_1. The value ϵ_1 inferred from this star is then $\delta_1 - \beta_1$. For a star with $\alpha = 270°$, with latitude β_2, and declination δ_2, the value ϵ_2 is $\beta_2 - \delta_2$. A bias that is common to both δ_1 and δ_2 thus disappears when we calculate the mean value of ϵ.

When we combine the measurements of Hipparchus quoted from the two sources cited (Ptolemy, *c* 142 and Hipparchus, *c* −128), the values of longitude or right ascension are well distributed, and Hipparchus's instrumental bias makes a negligible contribution to the inferred obliquity. The measurements of Timocharis, Aristyllus, and Ptolemy, however, are badly biased; most of the longitudes lie between 0° and 180°.* For safety, then, it is desirable to include instrumental biases in the analysis.

As matters stand, the only way to find an instrumental bias in ancient measurements of declination is to compare measured values with those calculated from modern theory. However, the comparison requires using a value for the obliquity, which is being treated as an unknown in this study. It would be possible to get around this difficulty by using an analysis in which the obliquity and all of the instrumental biases are treated as simultaneous unknowns, but luckily there is a simpler way. We see from equation (2) that the calculated declination is nearly independent of the obliquity if $\sin \lambda$ (or $\sin \alpha$) is near zero. Therefore we can find a good estimate of the declination bias in a set of observations by calculating values from equation (2) using a preliminary value of ϵ, comparing the calculated values with the measured ones, and weighting individual estimates by the factor $\cos^2 \alpha$ (the reader should remember that we use $\sin^2 \alpha$ for the weight when we are inferring the obliquity). Then we apply the estimated declination bias to the measured values before we use equation (2) again to infer the obliquity.

In principle, this is an iterative process, because the obliquity that we find is probably not the one that we use in estimating the bias. If necessary, then, we can use the new obliquity in order to estimate a new bias, and so on. In practice, the inferred bias is so insensitive to the obliquity that the first inference of the bias is more than adequate.

The observations of Timocharis and Aristyllus pose two minor problems. Ptolemy quotes 12 stellar observations by Timocharis and only six by Aristyllus. The Aristyllus sample is so small that one dislikes to treat it as an independent sample, particularly when two unknowns are to be inferred. Further, Timocharis and Aristyllus are contemporaries, and both observed in Alexandria when it was a young city and probably not liberally endowed with astronomers. Thus there is a good chance that they worked in cooperation, and I shall assume that they used the same instrument.

* I thank one of the referees for pointing this out to me. I originally carried out the analysis without using the instrumental biases.

Their declination of α Boötis is in error by about 0°·78. This is about a '3σ' value for ancient Hellenistic measurements. The overall sample obtained by combining the data from Ptolemy and Hipparchus is so large that it is not surprising to find one error this large. Because sin α was nearly zero for α Boötis at the time of the observation, the error has little direct effect upon the inferred obliquity. However, for the same reason, the error has a large effect upon the inferred instrumental bias and this plays indirectly into the obliquity. When the declination of α Boötis is used in calculating the instrumental bias of Timocharis and Aristyllus, the result is 0°·135; the plus sign means that this amount is to be added to the measured declinations. When α Boötis is omitted from the calculation, the bias drops to 0°·068.

Changing the instrumental bias for Timocharis and Aristyllus from 0°·068 to 0°·135 changes the final value of the obliquity by about 26″. This is a large amount by modern standards, but it is trivial in the context of ancient naked-eye observations. In the tables that I shall give later, I use 0°·068 for the instrumental bias; that is, I omit α Boötis from the estimate of the bias. However, I use it in the direct part of calculating the obliquity. By using the information that is supplied, the reader can easily use α Boötis in some other way if he wishes.

TABLE I

Stellar observations by Timocharis or Aristyllus and reported by Ptolemy

Star	Longitude* °	Latitude* °	Declination Measured °	Declination Corrected† °	sin ε
α Aql	269·596	29·394	$5\frac{4}{5}$	5·792	0·39929
α Aur	50·030	22·836	40	40·002	0·41033
α Boö	172·411	32·302	$31\frac{1}{2}$	31·500	0·47361
α CMa	72·675	−39·100	$-16\frac{1}{3}$	−16·351	0·40129
α Gem	78·505	9·891	33	33·001	0·40232
β Gem	81·776	6·506	30	30·000	0·40412
α Leo	118·160	0·365	$21\frac{1}{3}$	21·330	0·40726
α Lib	193·310	0·611	−5	−5·012	0·41642
β Lib	197·575	8·756	$1\frac{1}{5}$	1·191	0·39501
α Ori	56·918	−16·337	$3\frac{5}{6}$	3·825	0·40441
γ Ori	49·122	−17·108	$1\frac{1}{5}$	1·191	0·40297
α Sco	217·948	−4·270	$-18\frac{1}{3}$	−18·349	0·40023
α Tau	37·937	−5·627	$8\frac{3}{4}$	8·743	0·39744
η Tau	28·178	3·818	$14\frac{1}{2}$	14·495	0·40443
ε UMa	126·629	54·166	$68\frac{1}{2}$	68·512	0·39974
ζ UMa	133·322	56·262	$67\frac{1}{4}$	67·262	0·37995
η UMa	144·790	54·426	$61\frac{1}{2}$	61·510	0·40716
α Vir	172·048	−1·903	$1\frac{2}{5}$	1·391	0·40390

* Calculated for the epoch −290 January 1.

† Corrected for refraction, assuming that the declination was measured at culmination, but not corrected for instrumental bias. 0°·068 should be added to the values in this column in order to correct for the instrumental bias.

TABLE II

Stellar observations by Hipparchus and reported by Ptolemy

Star	Longitude* °	Latitude* °	Declination Measured °	Declination Corrected † °	sin ϵ
α Aql	271·863	29·386	$5\frac{4}{5}$	5·791	0·40123
α Aur	52·270	22·838	$40\frac{2}{5}$	40·401	0·40036
α Boö	174·653	32·193	31	30·999	0·43353
α CMa	74·888	−39·135	−16	−16·021	0·40206
α Gem	80·739	9·906	$33\frac{1}{6}$	33·166	0·39975
β Gem	83·989	6·520	30	29·998	0·39994
α Leo	120·390	0·374	$20\frac{2}{3}$	20·662	0·40123
α Lib	195·547	0·592	$-5\frac{3}{5}$	−5·615	0·40347
β Lib	199·814	8·739	$0\frac{2}{5}$	0·388	0·39828
α Ori	59·161	−16·315	$4\frac{1}{3}$	4·323	0·40252
γ Ori	51·364	−17·087	$1\frac{4}{5}$	1·789	0·40115
α Sco	220·187	−4·291	−19	−19·024	0·40131
α Tau	40·180	−5·616	$9\frac{3}{4}$	9·742	0·40180
η Tau	30·418	3·834	$15\frac{1}{6}$	15·161	0·39444
ϵ UMa	128·903	54·178	$67\frac{3}{5}$	67·610	0·39021
ζ UMa	135·602	56·272	$66\frac{1}{2}$	66·510	0·37733
η UMa	147·054	54·425	$60\frac{3}{4}$	60·758	0·39640
α Vir	174·286	−1·912	$0\frac{3}{5}$	0·588	0·40181

* Calculated for the epoch −128 January 1.

† Corrected for refraction, assuming that the declination was measured at culmination, but not corrected for instrumental bias. 0°·049 should be subtracted from the values in this column in order to correct for the instrumental bias.

TABLE III

Stellar observations made by Ptolemy

Star	Longitude* °	Latitude* °	Declination Measured °	Declination Corrected † °	sin ϵ
α Aql	275·592	29·374	$5\frac{5}{6}$	5·826	0·40117
α CMa	78·509	−39·193	$-15\frac{3}{4}$	−15·767	0·40339
α Gem	84·395	9·932	$33\frac{2}{5}$	33·401	0·40021
β Gem	87·611	6·542	$30\frac{1}{6}$	30·166	0·40100
α Leo	124·038	0·387	$19\frac{5}{6}$	19·830	0·40176
α Lib	199·218	0·561	$-7\frac{1}{6}$	−7·179	0·40727
β Lib	203·488	8·710	−1	−1·010	0·39775
α Ori	62·832	−16·279	$5\frac{1}{4}$	5·242	0·40669
α Sco	223·867	−4·325	$-20\frac{1}{4}$	−20·270	0·40159
α Tau	43·856	−5·598	11	10·994	0·40570
ϵ UMa	132·624	54·199	$66\frac{1}{4}$	66·262	0·39788
ζ UMa	139·333	56·288	65	65·011	0·43527

* Calculated for the epoch 137 July 20.

† Corrected for refraction, assuming that the declination was measured at culmination, but not corrected for instrumental bias. 0°·007 should be subtracted from the values in this column in order to correct for the instrumental bias.

5. INDIVIDUAL ESTIMATES OF THE OBLIQUITY

Each measurement of a declination furnishes an independent estimate of the mean obliquity ϵ. Information relating to the individual estimates is summarized in Tables I–IV. Tables I, II, and III refer to the three groups of data quoted by Ptolemy and Table IV lists the data quoted by Hipparchus.

TABLE IV

Stellar observations by Hipparchus and reported by Hipparchus

Star	Longitude* °	Latitude* °	Declination Measured °	Declination Corrected† °	sin ϵ	$W/\sin^2 \alpha$
ι And	346·717	40·958	32	31·999	0·40474	1
σ And	351·029	31·500	25·2	25·197	0·40204	1
α Ara	235·374	−26·237	−44	−44·089	0·39263	1
α Ari	8·043	9·912	12·2	12·193	0·36236	1
τ Ari	23·812	2·400	12·2	12·193	0·42771	1
ι Aur	47·073	10·190	27	26·998	0·40432	1
γ Boö	167·927	49·594	$48\frac{3}{4}$	48·754	0·40409	1
α Car	75·626	−76·112	$-51\frac{1}{2}$	−51·774	0·41593	1
ϵ Cas	25·344	47·335	52	52·005	0·44586	1
α Cep	343·704	68·910	$54\frac{1}{2}$	54·505	0·40207	1
ι Cep	4·286	62·468	$55\frac{3}{4}$	55·756	0·40105	1
β Cyg	271·834	49·210	25° 20′	25·330	0·40574	1
ζ Cyg	303·715	43·850	23	22·996	0·40565	1
η Cyg	283·651	54·489	31	30·999	0·40648	1
κ Cyg	285·797	73·952	50	50·004	0·41278	1
β Dra	222.100	75·542	55	55·006	0·40306	1
γ Dra	238·374	75·217	53	53·005	0·40171	1
μ Dra	204·768	76·438	$55\frac{2}{5}$	55·406	0·45205	0
ϵ Lyr	259·188	62·598	41	41·001	0·37260	0
β Oph	235·771	28·128	7	6·991	0·42078	$\frac{1}{4}$
ζ Oph	219·620	11·641	$-3\frac{1}{2}$	−3·513	0·39612	1
η Oph	228·360	7·416	−10·2	−10·217	0·40011	1
ι Oph	221·033	32·796	15	14·994	0·42235	$\frac{1}{4}$
γ Peg	339·635	12·559	3·6	3·590	0·40382	1
α Per	32·566	29·893	40	40·001	0·39488	1
τ Psc	358·796	20·640	20	19·995	0·20316	$\frac{1}{4}$
ψ^1 Psc	353·892	13·270	12	11·993	0·18135	0
g Psc	359·312	21·873	20	19·995	0·37519	$\frac{1}{4}$
θ Sco	236·011	−19·364	$-37\frac{2}{3}$	−37·721	0·39329	1
μ Sco	226·639	−15·112	−31	−31·038	0·39430	1
β Tau	52·993	5·209	$23\frac{1}{2}$	23·496	0·39549	1
β UMa	109·602	44·909	66	66·009	0·39819	1
γ UMa	120·576	46·967	65	65·009	0·40400	1

* Calculated for the epoch −128 January 1.

† Corrected for refraction, assuming that the declination was measured at culmination, but not corrected for instrumental bias. 0°·049 should be subtracted from the values in this column in order to correct for the instrumental bias.

The first column in each table gives the designations of the stars. The stars are listed in the alphabetical order of the constellations to which they belong, and alphabetically by letter within a constellation. The second and third columns give the longitudes and latitudes, as calculated from modern data. For Table III, which lists the measurements made by Ptolemy, the epoch used in the calculations is 137 July 20. This is equivalent to the nominal beginning of the reign of Antoninus Pius in the Egyptian calendar, and it it is the epoch to which Ptolemy refers his star table. Ptolemy says that Hipparchus made his measurements 265 years before, so I use −128 January 1 for Tables II and IV, which list Hipparchus's measurements.* I found no specific statement about the epoch of Timocharis and Aristyllus. I use −290 January 1 in Tables I, which is consistent with lunar and planetary observations that Timocharis is known to have made.

The fourth column in the tables gives the measured declinations. When the declination is given in decimal form, the declination was originally written in the form of being greater than a stated value, as I described in Section 2. Hipparchus gives the declination of β Cygni as $25° 20'$ (Table IV). Otherwise, the writers give all values as an integer alone, or as an integer plus a simple fraction, and the form used by the writers is the form that appears in the tables.† This practice gives the reader some idea of the precision implied by the quoted values.

The fifth and sixth columns give, respectively, the measured declinations after correction for refraction only, and the inferred values of $\sin \epsilon$ after correction for both refraction and bias. Table IV contains a seventh column labelled $W/\sin^2 \alpha$. The values in this column are the weights assigned in Section 2 on the basis of problems with the text; these problems appear only in the writing of Hipparchus. When $W/\sin^2 \alpha$ equals unity, the weight given to an observation is just $\sin^2 \alpha$. Otherwise, the weight is the product of $\sin^2 \alpha$ by the number listed in the seventh column.

6. CONCLUSIONS AND DISCUSSION

The individual estimates of $\sin \epsilon$ in each of Tables I–IV can be combined into four estimates that apply at specific epochs. The four composite estimates are summarized in Table V. The upper part of the table lists the observers, the approximate epoch of each composite estimate, the values of ϵ derived from the composite estimates of $\sin \epsilon$, and the estimated standard deviations. The standard deviations are based upon the scatter of the values in each table.

Although the four estimates in Table V apply at different epochs, we may use their weighted mean as the best estimate that can be derived from all the data, to the accuracy that the variation of ϵ with time is linear over the time span covered in the table, provided that we also use the weighted mean epoch. This gives us

$$\epsilon = 23° 43' 06'' \pm 3' 19'' \text{ at the epoch } -114. \tag{3}$$

For comparison, the value derived from equation (1) (Newcomb's expression) is

$$\epsilon = 23° 42' 35'' \text{ at the epoch } -114. \tag{4}$$

* Hipparchus probably drew up his star table after he discovered the precession of the equinoxes. In the work being quoted here, he seems to be unaware of the precession, and thus he may have written the work some years before −128. The exact year of the measurements does not matter for the purpose of this paper.

† In a few cases, the writer uses a sum of fractions, such as $\frac{1}{2}+\frac{1}{4}$. I have changed the sum into a single fraction, $\frac{3}{4}$ in this case.

TABLE V

Summary of the estimates of the obliquity

Observer	Approximate epoch	ϵ	Estimated standard deviation
Timocharis or Aristyllus	−290	23° 46′ 02″	8′ 30″
Hipparchus*	−128	23° 38′ 03″	5′ 4″
Hipparchus†	−128	23° 49′ 02″	6′ 19″
Ptolemy	137	23° 43′ 40″	8′ 42″
Mean	−114	23° 43′ 06″	3′ 19″
Newcomb's value	−114	23° 42′ 35″	—

* From the values quoted by Ptolemy (*c* 142).
† From the values quoted by Hipparchus (*c* −128).

The agreement between equations (3) and (4) is quite good.

I mentioned in Section 3 that the method of analysis is an iterative one. We start by assuming a value of the obliquity (more accurately, of the rotation of the ecliptic) to use in calculating the values of β needed in equation (2). We then use equation (2) in order to obtain a new estimate of the obliquity, and so on. I began the process by using Newcomb's theory of the obliquity, and the first iteration verified this theory to high accuracy. Therefore further iteration is not needed.

Including the instrumental biases changes the estimate of the obliquity by only about 5″. In spite of this small change, including the biases has three useful effects. First, it increases the confidence in the analysis. Secondly, when the bias was omitted, the estimates of the obliquity from the two samples of Hipparchan data in Table V differed by about 16′, while including the bias reduces the discrepancy to about 11′. Thirdly, including the bias lowers the residuals and thus increases the confidence level of the final estimate of ϵ.

The instrumental biases have no direct astronomical importance, but they have historical interest. They are listed for each sample in a note at the bottom of the corresponding table, with the same bias being used for both sets of observations made by Hipparchus. The biases range in magnitude from 0°·007 (about 25″) to 0°·068 (about 4′·1). The value of 25″ must be an accident; it does not seem possible that the techniques of the time were this accurate.

All of the estimates in Table V are less than 23° 51′ 20″, which is the value measured by Eratosthenes and 'confirmed' by Ptolemy in his hoax. Thus there is little question that 23° 51′ 20″ is too large at any epoch involved in Table V.

In an earlier work (Newton 1972), I estimated ϵ at the epoch 880 from a body of Islamic data with dates from 832 to 970. The estimate agrees with Newcomb's theory within 11″, and the standard deviation of the estimate is 20″. When a body of data of doubtful quality is omitted, the agreement is within 2″, with a standard deviation of 3″.

Thus the ancient and medieval data leave room for a correction to Newcomb's theory as large as about −0″·25T, which is the amount that Duncombe & Clemence (1958) suggest, but they seem to rule out any error that is much larger.

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