MEDIEVAL ASTRONOMICAL CONSTANTS

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All the data for these astronomical constants were taken from *Caesar's Astronomy* by Peter Apian (*Astronomicum Caesareum per Petrum Apianum*), published at Ingolstadt in A.D. 1540, a translation of which has been in course of preparation for nearly two years.

Apian's purpose in writing this book is well stated in his preface:

> We see how much mathematics are hated because of their seeming difficulty. We tried to simplify and began to plan by what new methods the remedy would be found and be a help to those, whom arithmetic has baffled up to now.
>
> Astronomy does not repel. It asks to be understood. If the universal theory of the heavens is reduced to instruments without the use of numbers and calculations, it would be a great help to those who have studied long.

Apian succeeded wonderfully in his task. At only two places in this book does he give any mathematics. In chapter xx, as an addendum "An important document and exceedingly useful one for astronomical work," he gives the mathematical way of computing the times of aspects, as an alternative to his graphical method. It involves nothing more complicated than long division. In chapter xl he gives a more accurate way of determining times:

> If you want the time more accurately, the daily motion of the Moon on the desired day must be taken, from which you will find the hour very easily by the rule of proportion.
>
> In the first place set the daily motion, in the second place 24 hours; in the third you put the distance of the critical time from noon; then, as the daily motion is to 24 hours, the distance is to what? With the rule completed, the quotient gives the hours and minutes.

However, some figures had to be given. They are mostly in the form of tables, one position for each hundred years, and from these tables it has been possible to determine the constants used by Apian.

The epoch 1540 has its special interest. The Ptolemaic system of crystal spheres, revolving around a stationary Earth, each with its own eccentric and epicycle, was at its height, soon to receive its first blow by the publication of Copernicus' *De revolutionibus orbium coelestium* in 1543.

Caesar's Astronomy gives descriptions of how to determine the positions of astronomical bodies by graphical means and the applications of astronomy to astrology, history, church calendars, and for the use of doctors.

It was dedicated to the Caesars, Charles V, Emperor of Rome, and his brother Ferdinand, King of Spain, etc., and their nativities are used throughout for examples.

This work was designed to be of more than ephemeral use. It was to be good for an aeon or two and a liberal interpretation may be placed on that expression.

Figures are available for determining the positions of the planets back to the beginning of the world and as the earliest date set for that event was by Alphonsus "6984 years & 121 days from Adam to Christ," they are carried back to the next preceding century, namely 7000 B.C.

Forward, probably for the sake of symmetry, they are carried to A.D. 7000, so we may have the comforting feeling that Apian's tables will still be usable 5000 years hence.

Apian achieved his purpose by the use of "instruments." These were of two classes.

1. Dial instruments. These are of extraordinary ingenuity. The backs are the leaves of the book. Superimposed on them are revolvable paper dials. Some instruments have as many as five dials, and up to three silk threads to serve as fiducial lines.

2. Polar co-ordinate charts. These are worked with a thread and "pearl" (bead marker).

Without working models to illustrate them any description would fall short and serve no purpose, so, quoting from Apian, "Counselling brevity, I prudently omit them."

The mean positions of the planets' longitudes, etc., are given for the beginning of each century in tables and have made it possible to determine the constants used by Apian, and so their accuracy, by comparing them with similar figures computed from the constants given in a modern *Nautical Almanac*.

The procedure has been practically the same in every case.

The modern figures for one year were multiplied by 100, corrections made for the Julian year and precession (if needed), and the difference of the planet's position in longitude determined in signs, degrees, minutes, and seconds, so that they could be compared directly with the differences for the same period, determined from Apian's tables.

The variations between the "century differences" are shown in two ways.

1. The angular error, measured in degrees, minutes, and seconds, and marked + or —, on the assumption that the modern determinations are correct and Apian's accordingly in error.

2. The time error, giving in days, hours, and minutes, the length of time beyond, or less than, a century, that it would take for the planet moving on Apian's constants to reach the position computed from modern data.

PRECESSION

Apian divided this motion into two parts:

1. *The movement of the ninth sphere under the tenth sphere,* which was his way of expressing the relative movements of the equator and the ecliptic.

The star map in chapter iv has a scale of years adjacent to the zodiacal scale, from which measurements of the movement could be made. For 10,000 years it scaled 73° 30′.

The table in chapter xii for determining the longitude of the Sun contains its position at the beginning of each century.

For 10,000 years the difference was 73° 28′, corresponding to an annual difference of 0′ 26.″45.

2. *Trepidation.*—This was an oval movement, completed in 7,000 years, but with the oval carefully arranged so that from 1000 B.C to A.D. 1000 the movement was practically identical with (1), but from then on the curve became more pronounced and reached the major axis at A.D. 1750, whence it started to go backward, so that from A.D. 2500 to A.D. 4500 it would practically neutralize the effect of precession.

Trepidation can hardly be called a constant, but for a long period it was nearly so at 7° per 1,000 years.

For the period 1000 B.C. to A.D. 1500, trepidation scaled 16°, corresponding to an annual difference of 23″.04.

Summarizing:

	1 year	100 years
Sun's motion in longitude	26″.45	44′ 05″
Trepidation	23.04	38 24
Sum of both motions	49.49	1° 22 29
At A.D. 250 from *Nautical Almanac* figures	49.89	1 23 09
Difference	− 0.40	40″
	+	295 days

With the single exception of trepidation, Apian's constants for motions were absolutely constant. They were the same between 7000 B.C. and 6900 B.C., for the same years A.D. and between 0 and 100 A.D.

Length of the Year

This can be deduced from the chart in chapter xxxix for the determination of the vernal equinox.

In 1540 the Julian calendar was in use, with one leap year in every four years, which resulted in a continuous recession of the vernal equinox.

Scaling from the chart showed a recession of 7 days and 11 hours in 1,000 years, which is equivalent to an annual recession of 10 minutes 44 seconds, and this corresponds exactly with the length of the year as determined by Alphonsus.

In chapter xxxviii the following determinations of the length of the year are quoted:

	Days	Hours	Min.	Sec.
Hipparchus	365	0	25	12
Julius Caesar	365	6	0	0
Ptolemy	365	5	55	22
Alphonsus, adopted by Apian	365	5	49	16
Nautical Almanac, computed for 1540	365	5	48	47
Difference	...	.	..	+29

Length of Summer

In chapter xxxix there is this interesting statement,

> If you add three days, 0 hours & 42^{m} to the number of days in March, on which the equinox occurs, you will have the day, hour & minute of the autumnal equinox, which occurs in September.

An examination of recent *Nautical Almanacs* disclosed that the time to be added now is 2 days, 10 hours, and 17 minutes. This is not a constant, but has varied between $2^d\ 10^h\ 07^m$ and $2^d\ 10^h\ 39^m$ in the past ten years.

The length of summer was determined for 1856 and 1786 by reference to the *Nautical Almanacs* for those years, and Mr. James Robertson, Director of the *Nautical Almanac* at Washington, kindly computed the mean interval for 1540, reporting it as 186.535 days. There are 184 days between March 1 and September 1, so this corresponds to $2^d\ 12^h\ 40^m$, and these four determinations fall on a smooth curve.

Apian's figure of $3^d\ 0^h\ 42^m$ differs from Mr. Robertson's by 12 hours and falls well off the curve.

This is the only mathematical error of magnitude that has been detected in the whole book. The figure $3^d\ 0^h\ 42^m$ was given twice and was the same in both places, so it is unlikely that it was a mere printer's error.

No apparent reason for this error has been found. Apian showed a decided sense of humor, but it is hardly reasonable to suppose that he was perpetrating a joke on himself, when he listed in the index under the letter *A*: "Abominable error concerning the vernal equinox, chapter ii."

Equation of Time

The equation of time was computed for 1540 in the usual form, but as Apian's chart gives it another way, always additive to apparent time, the modern style was converted into the medieval by adding 15.6 minutes to each determination. The dates given are for the Julian calendar.

It was found that the extremes agreed well. The constant to be added was chosen to make zero on January 31, which was Apian's zero date. The May maximum agrees within half a minute, though slightly off in date. However, in June discrepancies up to 3.75 minutes occur, which diminish gradually up to October, and the close concordance is maintained from then to near the close of the year. From December 30 to January 21 there is again an appreciable divergence.

The probabilities are that, with the tendency toward uniform motion in a circular orbit, which was favored by Ptolemaic astronomers, Apian did not allow properly for eccentricity and certainly the greatest discrepancies occur when the Sun is in apogee (as Apian would have expressed it).

EQUATION OF TIME FOR 1540
All additive to apparent time

Date of equinox, March 10.886
Date of perihelion, December 16.535

Date	Minutes: Apian	Minutes: *Nautical Almanac*	Apian's Differences
Jan. 1	7.0	6.8	+0.2
11	4.75	3.3	+1.45
21	1.25	0.9	+0.35
31	0	0	0
Feb. 11	0.65	0.6	+0.05
21	2.5	2.1	+0.4
Mar. 2	5.0	4.7	+0.3
11	7.85	7.9	−0.05
21	11.15	11.3	−0.15
31	14.4	14.5	−0.1
Apr. 10	17.1	17.3	−0.2
20	19.25	19.4	−0.15
May 1	20.6	20.4	+0.2
11	20.9	20.2	+0.7
21	20.2	19.2	+1.0
31	18.75	17.2	+1.55
Jun. 10	16.85	14.8	+2.05
20	14.9	11.9	+3.0
30	13.25	9.5	+3.75
July 11	12.0	8.6	+3.4
21	12.0	8.6	+3.4
31	12.75	10.0	+2.75
Aug. 10	14.4	12.4	+2.2
20	16.9	15.5	+1.4
30	20.1	19.2	+0.9
Sept. 10	23.9	23.1	+0.8
20	27.0	26.7	+0.3
30	29.8	29.7	+0.1
Oct. 10	31.8	31.8	0
20	32.7	32.8	−0.1
30	32.3	32.1	+0.2

Date	Minutes: Apian	Minutes: *Nautical Almanac*	Apian's Differences
Nov. 10	30.25	30.2	+0.05
20	27.25	26.9	+0.35
30	22.5	22.5	0
Dec. 10	17.5	17.5	0
20	12.4	12.3	+0.1
30	7.5	8.3	−0.8

Sidereal Periods of the Three Outer Planets

The sidereal periods can be determined from the tables given for finding their longitudes, as the mean positions are given for the beginning of each century. They are also given in slightly more convenient form in chapter xxxii for forecasting their mean conjunctions.

The figures are given for each century to the nearest minute, which was amply close enough for use with instruments that could be read, at best, to the nearest half-degree, but actually they were computed to about 5 seconds, so that the tables run with differences of, say 4′, for four or five centuries and then comes one with a 5′ difference and these intervals are then repeated.

For purposes of comparison, it was found to be easier to start with the modern figures. The sidereal periods were taken from the *Nautical Almanac* and the number of circuits made in 100 tropical years computed.

The number of completed circuits was then dropped and the fraction of a circuit converted into signs, degrees, minutes, and seconds.

To these were then added:

First, a small correction for the 0.78 of a day difference between 100 Julian years, consisting of 36525.00 days, and 100 tropical years, consisting of 36524.22 days, and

Second, the precession for a century near A.D. 1500; and the results so obtained were compared with the average secular differences in positions found in Apian's tables.

To quote Apian, "If this is not clear, Lo, I shall exemplify it to you."

Saturn

Sidereal period ....29.45772 years
Circuits per 100 tropical years........... 3.39470

	Signs	Degrees	Minutes	Seconds
0.39470 of a circuit................	4	22	05	31
Movement in 0.78 day...............	..	..	1	34
Precession for 100 years at A.D. 1500..	..	1	23	37
N.A. difference in position per century	4	23	30	42
Apian's average difference...........	4	23	28	05
Apian's error ....—	..	..	2	37

Similar computations for the other two outer planets gave:

Jupiter

Sidereal period ....11.86223 years

	Signs	Degrees	Minutes	Seconds
N.A. difference in 100 Julian years...	5	06	17	55
Apian's average difference...........	5	06	13	00
Apian's error ....—	..	..	4	55

Mars

Sidereal period ....1.88089 years

	Signs	Degrees	Minutes	Seconds
N.A. difference in 100 Julian years...	2	01	40	23
Apian's average difference...........	2	01	34	48
Apian's error ....—	..	..	5	35

Sidereal Periods of the Two Inner Planets

These were handled in the same way as for the outer planets, except that, as in the Ptolemaic system their mean motions were around the Earth, and these naturally were identical with that of the Sun, the positions of their epicycles, given in the table of arguments, or anomalies, had to be used.

The computations for *Venus* and *Mercury* follow:

	Venus	*Mercury*
Sidereal period (years)	0.61521	0.24085
Circuits per century	162.546	415.195

	Signs	Degrees	Minutes	Signs	Degrees	Minutes
Fractions of circuits...	6	16	34	2	10	12
Movement in 0.78 day..	..	1	13	..	3	11
	6	17	47	2	13	23
Difference of position of epicycle in century in table	6	18	14	2	12	18
Apian's errors	+..	..	27	—..	1	05

Precession was not included in these cases, as it had already been included in the movement of the Sun and so of the mean motions of these planets.

These angular errors, divided by the planet's motion in one day, give the time errors, expressed in days.

TABULATION OF THE ERRORS IN APIAN'S "SIDEREAL PERIODS IN ONE CENTURY"

		Degrees	Minutes	Seconds	Days
Saturn	—	..	2	37	+1.8
Jupiter	—	..	4	55	+1.0
Mars	—	..	5	35	+0.19
Venus	+	..	27	..	—0.28
Mercury	—	1	5	..	+0.26

The results shown in the two ways are apparently contradictory in that *Saturn* with the smallest angular error has the largest time error, while *Mercury* with the largest angular error by far has almost the smallest time error. This apparent contradiction is due, of course, to the big variation in speeds of travel.

THE MOON

The various movements of the Moon were derived from chapters xvii, xviii, and xxii.

In every case, the same procedure was adopted. The appropriate measurements were taken from the *Nautical Almanac* and the movements computed for 100 Julian years. The total number of circuits was then disregarded and the fraction converted into signs, degrees, and minutes, or into days, hours, and minutes, according to what was used by Apian.

"So that you can see the above more clearly, I shall explain it by an example."

Moon's Longitude

100 Julian years contain	36525.000 days
1 sidereal month contains	29.321661 days
Number of sidereal months in 100 Julian years	1336.851372
0.851372 of 360°	10^{z} 06° 29′ 38″
Add precession for 100 years at A.D. 1500	1 23 27
N.A. difference in longitude per century	10 07 53 05
Apian's difference	10 07 49 00
Apian's error	— 4 05

For the other constants, "as the method of working has been sufficiently explained, I prudently omit it."

Moon's Anomaly

Anomalistic month	27.554550 days
Number of months in 100 Julian years	1325.552404
N.A. difference in century	6^{z} 18° 51′ 55″
Apian's difference	6 18 42 15
Apian's error	— 9 40

Moon's Node

Daily movement "against the order of the signs".	3′.177
Movement in century	5.37222 circles
N.A. difference in century, "according to the order of the signs"	7^{z} 16° 00′ 00″
Apian's difference	7 15 50 40
Apian's error	— 9 20

CONJUNCTIONS OF SUN AND MOON

Synodical month	29.530588 days
Number of months in century	1236.85312
N.A. difference in century	$25^d\ 4^h\ 38^m\ 06^s$
Apian's difference	25 4 34 00
Apian's error	— 4 06

CONJUNCTIONS OF MOON AND NODE

Nodical month	27.212219 days
Number of months in century	1342.227916
N.A. difference in century	$6^d\ 4^h\ 51^m\ 14^s$
Apian's difference	6 4 42 40
Apian's error	— 8 34

It is noticeable that Apian's error in every case was negative "according to the succession of the signs." They thus tended to neutralize each other, when used for the location of eclipses, except for determining the time of the eclipse.

TABULATION OF THE ERRORS IN APIAN'S "LUNAR MOTIONS IN ONE CENTURY"

	Arc	Time
Longitude	— 4′ 05″	$+\ 7^m\ 26^s$
Anomaly	— 9 40	+17 45
Node	— 9 20	+ 3 days
Conjunction of Sun and Moon	$-\ 4^m\ 06^s$	
Conjunction of Moon and Node	— 8 34	

ECLIPSES

Chapters xxii to xxvii are devoted to eclipses, mostly to means for forecasting them, their time, duration, extent, etc.

As it is certain that the Sun & Moon are the principal sources in nature & the most powerful causes of events, both of births & deaths, and in truth, in the very best way transfer to nature their own strengths, virtues & accomplishments when least hindered, but when something obstructs them, they are hampered and weakened and destroy nature to the roots, so we judge that the consideration of these obstacles that cause distress will not be useless.

These chapters include solar eclipses, but only to a limited extent, for he remarks:

> You cannot forecast eclipses of the Sun to a nail's breadth without calculations (owing to diversity of aspects) as we have been able to for the Moon. . . . .

Apian's method, in general, is one of approximations and eliminations, trying for coincidences of conjunctions or oppositions of the Sun and Moon with nodes (Dragons' heads and tails), eliminating all times when eclipses cannot occur, repeating the operation on a more refined instrument, applicable only when the date is known approximately, and again with a third instrument to be sure that the solar eclipse will not occur by night, nor the lunar by day, when they could not be seen.

The later chapters give a polar co-ordinate instrument for converting the time of mean opposition into that of true opposition and a so-called dispositary and semidiametrical instrument for determining the extent to which the Earth's shadow will darken the Moon.

Having done all this by the use of five instruments and a chart, Apian shows in chapter xxviii how to determine all the elements of a lunar eclipse, and more accurately than before, by the use of a single instrument containing five polar co-ordinate curves.

Three illustrations are given:

Charles's eclipse	A.D. 1500 Nov. 5,	13^h 5^m
Ferdinand's eclipse	1502 Oct. 15,	12 11
Charles's Coronation eclipse	1530 Oct. 6,	12 31

These three eclipses are all recorded in the list of eclipses given in *Kanon der Finsternisse* by T. R. von Oppolzer. The same cannot be said of two of the three historical eclipses which follow.

Chapters xxix and xxx are devoted to correcting historical dates by means of coincidences with lunar eclipses, using the instances of:

1. The defeat of Darius by Alexander at Gangamela in 331 B.C.

2. The defeat of Perseus by Lucius Aemilius Paulus in Samothrace in 172 B.C.

3. The retreat of Nikias from Syracuse in 413 B.C.

In the first instance, the defeat of Darius, Apian was unable to decide between two eclipses about equidistant from the historian's date, and he remarks: "I cease now to judge which is correct and to rewrite histories completely, but I do indicate and build a path toward verifying them."

In the third instance, Nikias' retreat, he enters into a lengthy comparison of the Greek and Latin months to give the proper interpretation to Plutarch's account, and, despite the fact that he located a lunar eclipse at the right time of year and within three years of the supposed date, he finally decides that the reference was to a solar eclipse and so he was not able to determine it with his instruments.

Apian's conclusion to this instance follows:

> By a few examples of this kind, I wish to show, how, by the help of lunar eclipses, it is possible to free histories from errors, which are scattered through them, due to wrongly assigned dates.
>
> In the same way, sometime if God favor me, I shall treat solar eclipses in a special volume, in which careful work will be devoted to determine whether the date of the "Foundation of the City (of Rome)" is correct according to certain determinations which we have made.
>
> For this, eclipses of the Sun must be investigated principally and nothing can be truer than "true according to the unfailing succession of celestial motions."

In the second instance, the defeat of Perseus, he located the eclipse, which Livy tells us had been predicted by Sulpicius Gallus of the Roman army and its coming was announced in advance to the Roman army, who were thus duly warned, while the Macedonians, under Perseus, had no such warning and were panic-stricken by the eclipse.

This success can be considered an outstanding performance only for the year 1540. The tables that were used were made without the use of telescope, vernier, or chronometers. The instrument used to make the calculations was a circular slide rule, or more properly a series of circular slide rules of 12-inch diameter, but with them Apian was able to calculate the elements of this eclipse that occurred 1700 years before his time and the greatest error found was 1½ hours in the time of the eclipse.

The following table compares the elements that were given by both Apian and Oppolzer for the lunar eclipse of September 2 to 3, 172 B.C.

	Apian	Oppolzer
Middle of eclipse (Ingolstadt time)	$12^h\ 41^m$	$11^h\ 12^m$
Half-duration	1 44	1 37
Magnitude (12 = total)	12.5	11.4

The error of one and one-half hours in the time was a source of serious concern to Apian, as his computations did not agree with the time for the beginning of the eclipse given by Livy (and now confirmed by Oppolzer). Apian explains the discrepancy in this way:

> Livy said that the eclipse was seen from the second hour of the night to the fourth hour. It must be remembered that these hours were not counted from sunset, but from the end of twilight, as was the custom among the common people, whose opinions he probably followed, and particularly among the soldiers, whose observations were more for their armament than the firmament, who paid more attention to scars than to stars and whose thoughts were more on flagons than on dragons.
>
> So I think that the mistake was really due to the carelessness of the soldiers and nothing really persuades us, but that we should take 12 hours & 29 minutes, as it surely was, for the beginning of the eclipse (in Samothrace).

My thanks are due to A. W. Recht, professor of astronomy at the University of Denver, for his computation of the "Equation of time in 1540" and for his assistance in other calculations, also to my cousin, Captain Tristan Dannreuther, R.N., F.R.A.S., for having started me on this investigation.

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